Acoustic Metamaterials

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Abstract

The field of engineered materials with designed properties is expected to continue to grow in the future, and metamaterials are instrumental in allowing this freedom of design. Metamaterials, particularly acoustic, are still in the stage of infancy. Acoustic metamaterials are being explored theoretically, but there has been little headway on the experimental front. The design, development, and characterization of acoustic metamaterials will offer many opportunities in materials science. In this article, we review the basic physics of different kinds of acoustic periodic structures with special emphasis on locally resonant acoustic metamaterials. We first survey phononic crystals and then discuss localized resonances in intrinsic and inertial resonating structures of acoustic metamaterials. Finally, we present the ongoing efforts in realizing acoustic metamaterials with negative materials properties and discuss the implications of acoustic metamaterials.

Introduction

Metamaterials designed by engineering the underlying microstructure offer new opportunities in materials science and technology. Acoustic metamaterials, as a counterpart to electromagnetic metamaterials, have just begun to emerge. These are a subset of microstructured acoustic materials that fall into three nonexclusive categories: phononic crystals,[1–15] intrinsic acoustic metamaterials,[16–18] and inertial acoustic metamaterials.[19–28] Before the interest in acoustic metamaterials, phononic crystals were first investigated in the early 1990s as the analogue of photonic crystals. When constituent "atoms" of high impedance contrast with the matrix are arranged spatially on the order of the matrix acoustic wavelength, band folding due to Bragg scattering results in bandgaps[1,2] and other extraordinary phenomena.[3] However, a reliance on Bragg scattering makes phononic crystals unfeasible at low frequencies because of the long acoustic wavelengths, necessitating impractically large samples. In addition, phononic crystal media cannot be ascribed effective properties, such as acoustic impedance or index, that are a key to using them as "materials" when designing new structures and devices.

Acoustic metamaterials, also known as locally resonant sonic materials, provide a major step toward an effective medium description. By properly engineering resonators into each acoustic "atom," one can achieve a unit cell that is deep-subwavelength at the resonance frequencies, thus enabling multiple scattering to be treated in an average sense and effective properties such as mass density and bulk modulus to be ascribed to the material. Acoustic metamaterials can be further classified into intrinsic and inertial, depending on whether the size of the resonating elements is completely decoupled from the wavelength. In this review, we present a brief history of phononic crystals, followed by a discussion of recent advances in acoustic metamaterials.

Phononic Crystals

The term phononic crystal has been used to label many different periodic fluid, elastic, and combination structures. Here, the term acoustic phononic crystal (APC) is used to describe materials with elastic or fluid inclusions inside a (different) fluid matrix. When the matrix is an elastic solid, the material is called an elastic phononic crystal. The anomalous behavior in phononic crystals arises from interference of waves strongly scattered off the inclusions and transmitted through the inclusions and matrix. For the common case of effectively rigid inclusions in an APC (e.g., steel in air), the bandgaps appear when the layer spacing in the propagation direction is nominally one-half of the wavelength in the matrix fluid, causing destructive interference.

Aside from bandgaps,[7–9] one of the most interesting phenomena created by phononic crystals is focusing through negative refraction.[3–6] The focusing is not due to a negative refractive index; instead, it utilizes bands beyond the first Brillouin zone with eigenfrequency contours convex to the origin at specific frequencies.[3] This results in a range of incident wavevectors being focused in two dimensions[6] and three dimensions.[3] It is emphasized that Snell's law is valid, with the phase front direction undergoing normal refraction.

When the inclusion impedance and phase speed are greater than those of the matrix but not effectively infinite, additional interesting phenomena can occur.[10–13] The inclusions allow waves to pass through, making the field a complicated interference of scattered, inclusion-transmitted, and matrix-transmitted waves. This results in asymmetric transmission peaks and dips as a function of frequency, called Fano profiles.[14] The case where the inclusion material is very similar to the matrix material tends to yield no new phenomena, as the waves are only weakly scattered. However, when the phase speed of the inclusion material is significantly lower than that of the matrix material, local resonances can occur, resulting in acoustic metamaterials.

Acoustic Metamaterials

Acoustic metamaterials derive their unique properties from resonators contained within each unit cell. The general physics behind the unusual materials properties is straightforward. When a mechanical oscillator is driven through resonance, the phase delay of the "response" acceleration relative to the driving force abruptly changes from a small amount to nearly 180° as the displacement amplitude becomes large in low-loss materials. The large scattered and reradiated fields interfere constructively or destructively or cause anomalous phase shifts when added to the background input wave.

One of the key differences between locally resonant sonic materials and phononic crystals is dependence of the resonance frequency on the geometry, including lattice parameters and incidence direction. Phononic crystals require the wavelength to be on the order of the lattice constants in the propagation direction at the bandgap center frequency. However, each unit cell in an acoustic metamaterial contains its own mechanical oscillator. These unit cells are very small and have minimal crosstalk, leaving the individual resonator eigenfrequencies insensitive to lattice parameters and direction. The two methods of creating local resonances, including examples, are discussed next.

Intrinsic Acoustic Metamaterials

In intrinsic acoustic metamaterials, the inclusion phase speed is much lower than that of the matrix fluid.[16–18] One can understand the resonance phenomenon by considering the relative inclusion and matrix wavelengths at resonance. The lowest order eigenmodes of an inclusion nominally occur when the wavelength inside the inclusion reaches twice the inclusion size. Assuming that the lattice constant is on the order of the inclusion size, this means that the matrix wavelength is much larger than the unit cell when the inclusion resonances are reached. For this type of acoustic metamaterial, a material with a very low phase speed is needed. The most common choice is a soft silicon rubber,[15] which has phase speeds that are two orders of magnitude lower than those of typical solids.

Very few studies have considered inclusions consisting entirely of soft rubber. One numerical study utilized soft silicon rubber in an acoustic metamaterial,[16] and another used it in an elastic metamaterial.[29] Experimentally, an investigation using this material found a transmission resonance that was believed to be localized in the rubber.[30,31] However, as in other publications[23–25] considering this material, an additional heavy component was placed inside the soft material to create an inertial acoustic metamaterial.

Inertial Acoustic Metamaterials

Instead of requiring materials with very low phase speeds, local resonances can also be created through inclusions of two or more components that function as mass–spring–damper oscillators in each unit cell.[19–28] Two resonators used in recent inertial acoustic metamaterials are the Helmholtz resonator[19–22] and coated cylinders/spheres.[23–25] Helmholtz resonators consist of a cavity in a rigid material connected to the fluid matrix through a much narrower throat. A unit cell of such a construct is shown in Figure 1. The fluid in the throat acts approximately as a mass, whereas the compressible fluid in the cavity performs the function of a spring. With the appropriate throat and cavity dimensions, individual Helmholtz resonators can be made deep-subwavelength at resonance.

The concept of coated cylinders/spheres as resonators gained prominence in a report detailing an experimentally tested elastic inertial metamaterial. By coating heavy spheres with soft silicon rubber and encasing the coated spheres in epoxy, actual mass–spring–damper resonators were created, as shown in Figure 2.[30] This idea was extended in theoretical works in

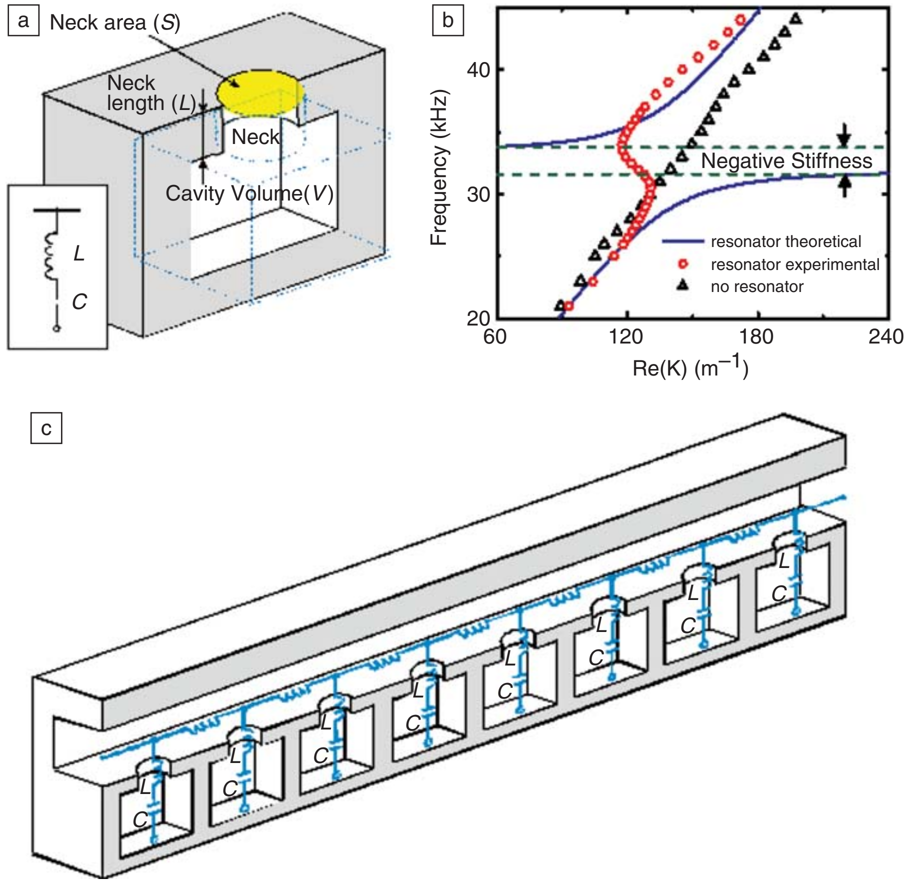

Figure 1. (a) Cross-sectional view of a Helmholtz resonator. A cavity is carved out of a rigid material (gray) and connected to the outside through a neck. Forces applied to the neck area *S* drive neck fluid approximately as a mass into the cavity, which compresses like a spring. The inset illustrates the analogy between a Helmholtz resonator and an *LC* (or inductor–capacitor) circuit. (b) Dispersion relationship for the one-dimensional chain of Helmholtz resonators shown in part (c).[19] The region where the real wave vector (inverse wavelength) decreases with increasing frequency marks the resonance region where the bulk modulus is negative.

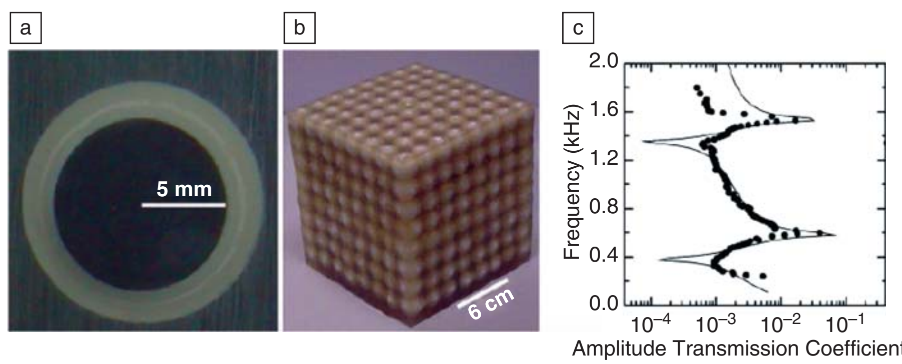

Figure 2. (a) Cross section of an inertial acoustic metamaterial unit cell consisting of a silicon-rubber-(white) coated hard sphere (black) embedded in an epoxy matrix (gray).[30] The hard sphere oscillates back and forth in the soft rubber, forming the resonant element. (b) Unit cells arranged in an 8 × 8 × 8 block of metamaterial that was experimentally tested. (c) Transmission dips associated with local resonances.

which the epoxy matrix is replaced by a dense fluid, and the same low-frequency resonances were found.[23,24]

Negative Acoustic Properties

The goal of creating acoustic metamaterials is to go beyond the bandgaps,[7] focusing,[3–6] and waveguiding[32,33] found in phononic crystals and create new effective material properties. We focus here on the development of negative effective acoustic properties.

Negative Bulk Modulus

Bulk modulus is defined through the constitutive relation between pressure and volume:

$$B = -V\frac{\partial p}{\partial V}. \tag{1}$$

In Equation 1, B is the bulk modulus, V is the volume, and p is the gauge pressure. Negative bulk modulus occurs when a unit cell, on average, is expanding when the applied gauge pressure is positive, which is reminiscent of the spherically symmetric radial oscillation of a monopole resonator above its resonance frequency. Spheres have natural monopole resonance modes, and soft rubber spheres in water have been numerically shown to cause a negative bulk modulus over a small frequency window above resonance.[16]

Subwavelength Helmholtz resonators radiate in a hemispherical pattern but can also be, in effect, monopoles in narrow one- and two-dimensional waveguides, where the shadow region does not exist. Experimental evidence (Figure 1c) has been presented asserting that a one-dimensional chain of Helmholtz resonators has a negative group velocity due to a negative bulk modulus.[19] Many groups have subsequently numerically studied geometric and design variations and developed models to assist in interpretation.[20–22]

Negative Mass Density

Effective mass density is defined through Newton's second law as

$$\rho = \frac{1}{a}\frac{F}{V}, \tag{2}$$

where ρ is the mass density, a is the acceleration, F is the total force acting on the element, and V is the volume of the element. Negative mass density implies that the average acceleration of a unit cell is opposite to the driving force.

Inspired by experiments on coated spheres in an elastic metamaterial (Figure 2),[30] negative mass density has been shown numerically in acoustic metamaterials.[24] The interpretation of the coated-sphere inertial elastic metamaterial as a mass–spring–damper oscillator naturally leads to the explanation of transmission dips as resulting from the core oscillating strongly out of phase with the driving force on each epoxy unit cell. Assuming that the core oscillation is strong enough to dominate the average acceleration, it can be shown that the real part of the mass density is negative.

Negative Index

In the case of purely real properties, negative refractive index in acoustics as defined in Equation 3 requires both the real mass density and the real bulk modulus to be negative:

$$n = \sqrt{\frac{B_0\rho}{\rho_0 B}}. \tag{3}$$

In Equation 3, n is the index of refraction, B_0 is the reference bulk modulus, and ρ_0 is the reference density. Assuming positive reference materials properties, when the metamaterial bulk modulus and density are both negative, the term under the square root is positive. However, as in the electromagnetic case, it can be shown through causality that the negative square root should be chosen.[16] In this case, in addition to the refraction being negative, the evanescent waves will be enhanced across a slab of negative index, leading to subwavelength imaging.[34] A negative-index acoustic metamaterial using the intrinsic resonances of soft spheres has been proposed.[16] Assuming sufficiently small losses, the monopole and dipole resonance frequency ranges overlap, resulting in a negative modulus and density. In the elastic metamaterial field, a negative-index elastic metamaterial design has been proposed using two separate resonators, one for negative density and one for negative bulk modulus.[18]

Effective Property Extraction

The method of extracting effective properties depends on the simulation technique for numerical studies and measurable variables in experimental studies. For the multiple-scattering technique, the multipole scattering coefficients are related, to first order, to physical properties. In acoustic systems, the first-order monopole coefficient can be related to the bulk modulus, whereas the second-order dipole coefficient is related to the mass density.[35]

For other techniques and experiments that solve for the fields, one can use equations relating the reflection and transmission coefficients of a slab of material to the effective refractive index and impedance of the slab.[36,37] For a beam obliquely incident on the layer of metamaterial, one can measure the transmitted beam displacement or internal field and the intensity of the reflected beam to infer the refractive index and impedance of the slab.

Future Directions

Many uses have been proposed for acoustic metamaterials and acoustic phononic crystals (APCs). The most obvious are as noise-damping and -shielding materials, as single negative properties imply strong damping and bandgaps imply that transmission is forbidden. Experimental demonstrations have shown that the mass density law is broken by a large amount within very narrow frequency windows.[28]

The other natural proposed application of acoustic metamaterials and APCs is as lenses. APCs have already been shown to focus pressure waves at very specific wavelengths and could be used immediately.[3–6] However, because this focusing is scattering-based, only far-field components are focused, and the resolution is no better than that of normal lenses. Negative-index lenses have the potential to circumvent this constraint and break the diffraction limit by focusing near-field evanescent pressure waves as well.

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