Comment | Vol. 35, No. 4 / April 2018 / Journal of the Optical Society of America A 591

Journal of the Optical Society of America A
OPTICS, IMAGE SCIENCE, AND VISION

Comparison of different theories for focusing through a plane interface: comment

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Received 15 August 2017; revised 31 October 2017; accepted 6 November 2017; posted 30 January 2018 (Doc. ID 304901); published 21 March 2018

In light focusing through a dielectric interface, Wiersma *et al.* [J. Opt. Soc. Am. A 14, 1482 (1997)] claim that the Debye–Wolf diffraction theory and the *m*-theory predict axial focal fields with "little difference." We found a possible mistake of using an inaccurate apodization factor in the *m*-theory integral. Here we correct the apodization factor, which then leads to better agreement on axial intensity distributions between the two theories than reported.

OCIS codes: (260.1960) Diffraction theory; (180.0180) Microscopy; (080.1510) Propagation methods.

https://doi.org/10.1364/JOSAA.35.000591

Light focusing into a dielectric interface occurs in many important applications, including optical microscopy, optical trapping, and laser direct writing. Among several vectorial diffraction theories, the Debye–Wolf integral at high numerical aperture (NA) (or high Fresnel number regime) [1] may be most widely employed in such applications. Wiersma *et al.* [2] used the *m*-theory diffraction integral from which the predicted axial distribution has "little difference" with the high NA Debye–Wolf solution. Such a small but certain difference that exists has been believed true and cited in many application studies. However, we rectify that if correct apodization (introduced as an amplitude factor in [2]) is used in the *m*-theory integral, both theories in fact produce practically the same normalized axial fields in high NA focusing when the interface is not too close to the index-matched focus ($z = 0$).

We notice that the different profiles of the reported axial intensity were caused by an incorrect apodization factor originally derived in [3] based on conservation of energy and used in [2–4]. An energy flowing through a surface (A) is given by $\int \vec{S} \cdot d\vec{A}$, which sums time-averaged energy flux S $(= \frac{1}{2} Z^{-1} |\vec{E}|^2$, where $\vec{E}$ depicts a complex amplitude of electric fields and Z a medium impedance) projected to the surface normal. In light focusing as illustrated in Fig. 1, when a spherically converging wave, E_1, right after the exit pupil propagates immediately before the planar dielectric interface (z_1^+), the associated apodization can be derived by equating energies flowing through each surface (neglecting the same impedance in the n_1 medium):

$$\iint_\Omega |E_1(\theta_1)|^2 f^2 \mathrm{d}\Omega = \iint |E_{z_1^+}(\rho)|^2 \cos\theta_1 \rho \mathrm{d}\rho \mathrm{d}\phi, \quad \textbf{(1)}$$

where $d\Omega = \sin\theta_1 d\theta_1 d\phi$. Note that the incident ray vectors to the plane interface angled to the surface normal ($-\hat{z}$), thus appending a projection factor $\cos\theta_1$. Transforming the cylindrical integral coordinate on the right-hand side of Eq. (1) to spherical coordinate by $\rho = z_1 \tan\theta_1$ (here, $z_1 = f - d$ in [2]) and comparing both-side integrands, the apodization factor is drawn as

$$A(\theta_1) = \frac{E_{z_1^+}(\theta_1)}{E_1(\theta_1)} = \frac{f \cos\theta_1}{z_1}, \quad \textbf{(2)}$$

which is simply a distance ratio of each field location with reference to the index-matched focus ($z = 0$), i.e., $f : z_1 \sec\theta_1$ like the inverse square law of intensity. This amplitude factor differs by $\sqrt{\cos\theta_1}$ from Eq. (17) in [2], or Eq. (16) in [3], where the inner product nature in calculating a total energy was probably missed.

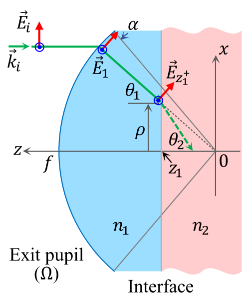

Fig. 1. Aplanatic focusing through a dielectric interface at $z = z_1$. f, focal length; α, semi-aperture angle; (f, θ, ϕ), spherical pupil coordinate; (ρ, ϕ, z_1), cylindrical interface coordinate.

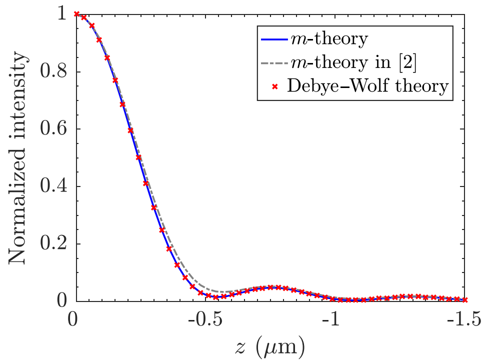

Fig. 2. Light focusing into an index-matched medium (oil $n_1 = n_2 = 1.522$) at $z_1 = 50$ µm. The m-theory with correct apodization yields the same axial distribution by the Debye–Wolf theory. $f = 1.8$ mm, NA = 1.4, vacuum $\lambda_0 = 488$ nm.

Then the axial distribution by the m-theory diffraction integral, if a x-polarized, uniformly incident light is aplanatically focused, is corrected as

$$E_x(z) = \frac{f}{2} z_1 (z - z_1) \int_0^\alpha \left(\frac{1}{s^3} - \frac{ik_2}{s^2} \right) \exp(ik_2 s + ik_1(f - t)) \times (\tau_s + \tau_p \cos\theta_2) \frac{\tan\theta_1}{\sqrt{\cos\theta_1}} \mathrm{d}\theta_1, \tag{3}$$

where s, t, τ_s, τ_p are defined in [2] with wave numbers in each dielectric medium as k_1 and k_2. Here, the near-field term s^{-3} can be often neglected if $sk_2 \gg 1$.

We verify the apodization factor, Eq. (2), by numerically comparing axial intensity distributions in an index-matched case ($n_1 = n_2$) in Fig. 2. The m-theory result, Eq. (3), and the Debye–Wolf solution, Eq. (10) in [2], agree perfectly with each other, which confirms that our apodization factor is correct. The inexact apodization (Eq. (17) in [2]), on the other hand, results in the broader main-lobe of axial intensity when $n_1 = n_2$.

Axial intensity distributions under index-mismatch circumstances are compared in Fig. 3. The corrected m-theory solution, Eq. (3), shows excellent agreement with the Debye–Wolf intensity in the glass/water interface. The inaccurate apodization in [2] gives rise to lower side-lobe intensity when normalized by the main-lobe peak. Even if normalized as done in [2], we checked that better agreement results. We also noticed that at 1.4 NA the main-lobe profiles predicted from both theories could be considered practically identical even for the interface being as close as $z_1 = 2$ µm, although as pointed out in [2] the approximated boundary field at the planar interface becomes less reasonable.

In conclusion, we corrected the inaccurate apodization factor in [2–4], based on the conservation of energy, associated with a spherically converging wave seen on a planar interface. The correct apodization derived was numerically validated by showing that in the index-matched focusing at 1.4 NA the on-axis intensity profiles from the m-theory integral evaluated on the planar interface and the Debye–Wolf integral evaluated on the spherical exit pupil are identical. In focusing through a planar interface, the normalized axial intensity distributions from both diffraction theories could be considered the same at higher Fresnel number regime if an interface is not too close to the index-matched focus.

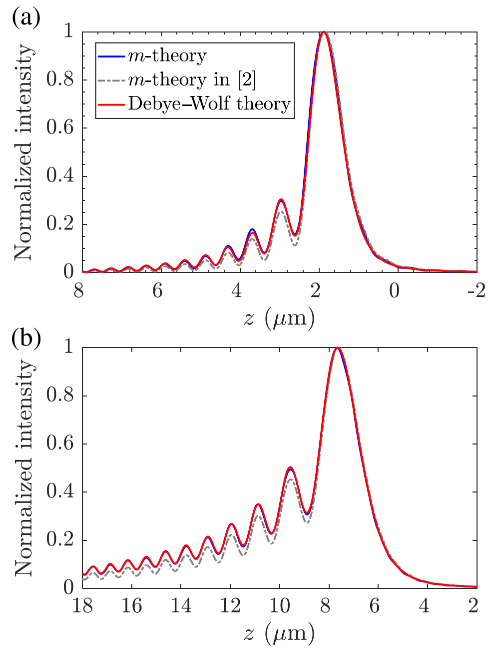

Fig. 3. Comparison of normalized axial intensity distributions if focused through a glass/water interface (1.522/1.337) at 1.4 NA. $\lambda_0 = 488$ nm, $f = 1.8$ mm. (a) $z_1 = 10$ µm and (b) $z_1 = 50$ µm.

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