PHYSICS

# Stable Casimir equilibria and quantum trapping

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**The Casimir interaction between two parallel metal plates in close proximity is usually thought of as an attractive interaction. By coating one object with a low–refractive index thin film, we show that the Casimir interaction between two objects of the same material can be reversed at short distances and preserved at long distances so that two objects can remain without contact at a specific distance. With such a stable Casimir equilibrium, we experimentally demonstrate passive Casimir trapping of an object in the vicinity of another at the nanometer scale, without requiring any external energy input. This stable Casimir equilibrium and quantum trapping can be used as a platform for a variety of applications such as contact-free nanomachines, ultrasensitive force sensors, and nanoscale manipulations.**

In 1948, Hendrik Casimir predicted that an attractive force occurs between two parallel, uncharged, perfectly conducting plates closely separated in a vacuum; this force has come to be known as the Casimir force (*1*). The effect arises from quantum fluctuation–induced temporary electromagnetic fields between the two plates (*2*). Electromagnetic modes between two plates are discretized so that the total intensity of fluctuation-induced electromagnetic fields between the plates is less than that in free space (*3*). Thus, the plates are pushed toward each other as a result of unbalanced electromagnetic pressure in the confined space (*4*).

The Casimir force between two mirror-symmetric objects of the same material has been proven to be always attractive, monotonically increasing as the separation decreases independently of the objects' shape, local dielectric function, and environment (*5*). No stable Casimir equilibria have been found to exist between electrically neutral objects composed of the same materials, regardless of whether their permittivities are higher or lower than that of the environment medium (*6*).

The attractive nature of the Casimir effect is detrimental for micro- and nanomechanical systems, resulting in irreversible adhesion (*7*–*9*) and frictional forces (*10*, *11*) as well as undesired aggregation of nanoparticles (*12*). The possibility of repulsive Casimir interactions has thus prompted researchers to pursue stable Casimir equilibria. The monotonically repulsive Casimir force can be achieved by embedding two objects of different materials in a fluid (*13*–*15*). However, the stable Casimir equilibria remain elusive. In this work, we address the question of whether Casimir equilibrium exists, meaning that Casimir forces can be repulsive at short separation distances and attractive at long distances.

Stable Casimir equilibria were predicted in theory by arranging one of the interacting objects enclosed by another (*16*, *17*) so that the surrounding repulsive Casimir forces could shroud the object at the center. This special topological requirement limits possible applications and also makes experimental verification extremely difficult. Because Casimir forces at large separations are mainly contributed by low electromagnetic frequencies and at small separations by high frequencies, a stable Casimir equilibrium could be realized if small frequencies contribute only attractive forces and large frequencies provide sufficient repulsive forces (*18*, *19*). Owing to difficulties in weak force measurement in liquid environments and the strict combination of materials, no experiment to date has verified this theoretical prediction, although indirect evidence has been found in interfacial premelting of ice (*18*). Other approaches associated with the design of specific geometries (*20*–*22*) were proposed, but these methods can produce Casimir equilibrium only along the axis of symmetry, leaving instability for displacements in other directions. Furthermore, although theoretical studies with exotic materials (*23*–*27*) or excited-state atoms (*28*) also suggest that it is possible to obtain stable Casimir equilibria, no experimental evidence has been demonstrated. In this study, we theoretically propose and experimentally demonstrate that stable Casimir equilibria can be

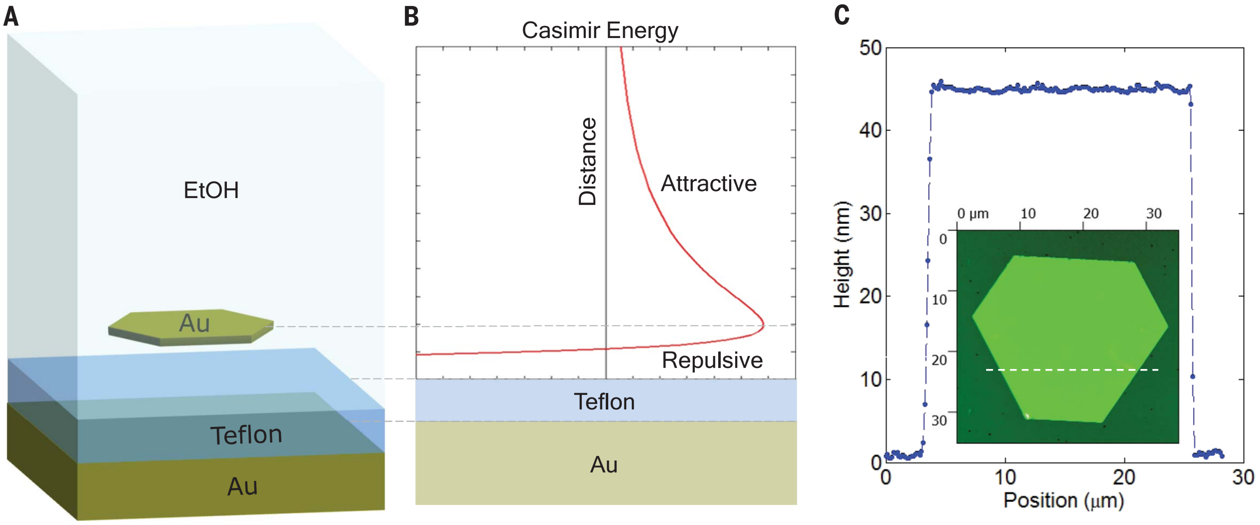

**Fig. 1. Stable Casimir equilibrium enabled by a low–refractive index coating layer.** (**A**) By coating a thin layer of Teflon on a gold substrate, a stable Casimir equilibrium is formed so that a gold nanoplate can be trapped at an equilibrium position in ethanol. (**B**) Casimir interaction energy between the gold nanoplate and the Teflon-coated gold surface. The Casimir force given by the derivative of the Casimir energy with respect to the distance is repulsive at short distances and attractive at long distances. (**C**) Thickness and surface profile of the gold nanoplate along the dashed line in the inset AFM image of the gold plate.

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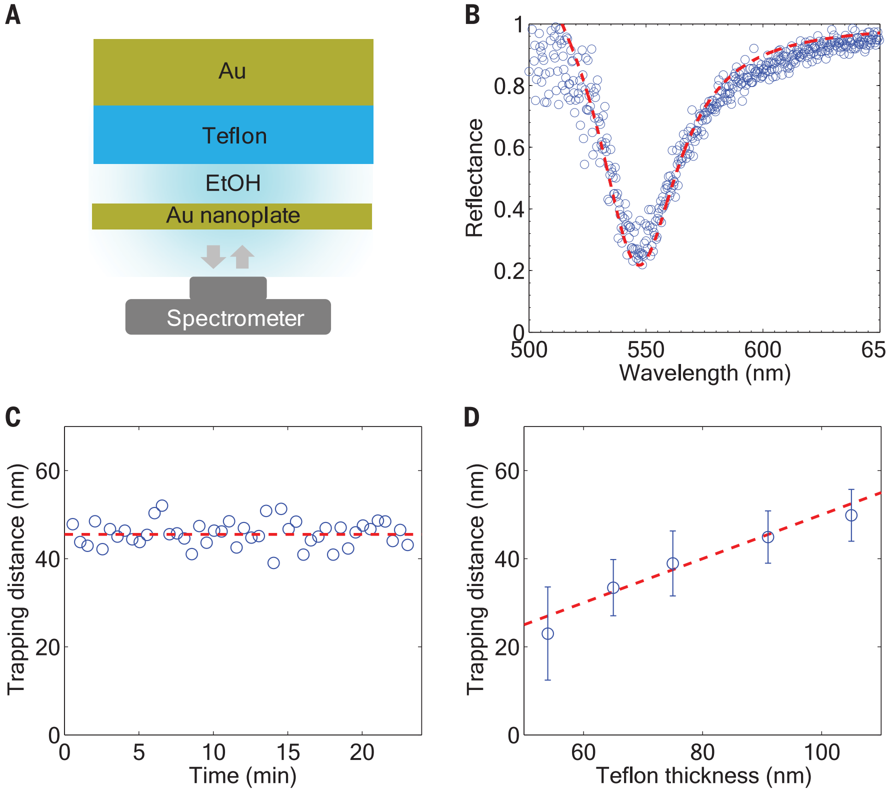

**Fig. 2. Trapping distance determined by the thickness of the Teflon film.** (**A**) By measuring the reflectance from an area of 5 μm in diameter at the center of the nanoplate (fig. S5), the distance between the gold nanoplate and the Teflon surface can be determined by the Fabry-Pérot resonance in the reflectance spectrum. (**B**) Reflectance spectrum for a sample with a Teflon thickness of 91 nm. The whole spectrum was taken within 0.05 s. The dashed line is the fitting reflectance where the trapping distance is the only fitting parameter. (**C**) Trapping distances of the gold nanoplate over 23 min. The dashed line is the theoretical prediction without considering the thermal variation. (**D**) Trapping distance versus Teflon thickness. The experimental results were averaged over multiple measurements of multiple trapped particles. Error bars show the variation of the average position of multiple trapped particles. Because the Teflon thickness is precisely measured using the ellipsometer and confirmed using AFM, none of the error bars show the Teflon thickness variation. The dashed line is the theoretical prediction calculated by eq. S1.

achieved by coating one high–refractive index object with a thin layer of a low–refractive index dielectric.

We designed an experiment on Casimir trapping in a fluid, between two gold surfaces as high-index media, one of which is coated with a low-index polytetrafluoroethylene (Teflon, DuPont) (Fig. 1A). The three materials were chosen carefully with a designed permittivity relation (fig. S1). Calculations show that the Casimir force between the gold nanoplate and the Teflon-coated gold surface is repulsive when these objects are near each other and attractive at longer distances (Fig. 1B). The gold nanoplates were chemically synthesized with a single-crystal hexagonal shape, a thickness of 45 nm, and a lateral size of ~25 μm (Fig. 1C) (*29*). The atomic force microscopy (AFM) tomography images show that the surface peak-to-valley roughnesses of the gold nanoplate, the gold substrate, and the Teflon film are less than 2 nm (Fig. 1C), 2 nm (fig. S2A), and 6 nm (fig. S2B), respectively. Such a nonmonotonic Casimir interaction between a gold surface and a Teflon-coated gold surface in ethanol solution is confirmed by direct force measurement using AFM (fig. S3). At the equilibrium position, the Casimir interaction energy reaches its minimum, creating a trapping potential. As a result, the gold nanoplate can be trapped near the surface at a certain distance. Moreover, the trapping distance is determined by the thickness of the Teflon coating (fig. S4).

Under an upright microscope, the gold nanoplates were clearly observed undergoing random Brownian motion in a given plane parallel to the surface without adhesion to the Teflon, which suggests that a strong repulsive Casimir interaction exists at a short distance between the gold nanoplate and the Teflon surface. When the devices were flipped upside down and transferred onto an inverted microscope (fig. S5), the gold nanoplates were still able to undergo random Brownian motion (movie S1 and fig. S6), confirming the existence of attractive Casimir interactions at long distances and, therefore, the trapping of gold nanoplates in the vicinity of the Teflon surface.

To quantify this stable Casmir equilibrium, we measured the reflectance spectrum from each gold nanoplate trapped in the vicinity of the Teflon-coated gold substrate (Fig. 2A). The 45-nm-thick gold nanoplate behaves like a low-reflectivity mirror. When the low-reflectivity plate is brought within a certain distance of the high-reflectivity 200-nm-thick gold substrate, the Fabry-Pérot resonance dip is observed in the reflectance spectrum (Fig. 2B). By fitting the measured reflectance spectrum, the trapping distance can be experimentally determined (fig. S7). Variation of the measured trapping distance is very small (σ ~ ±3 nm) (Fig. 2C), which indicates the existence of strong trapping force that provides a steep Casimir trapping potential. This trapping force provided by quantum fluctuation–induced electromagnetic fields is passive without any external energy input and can be much stronger than the optical trapping force where a high-intensity laser is needed.

As the Teflon thickness increased, the measured trapping distance increased proportionally to nearly half of the Teflon thickness (Fig. 2D). The good agreement between experimental results and theoretical predictions indicates that the trapping potential is solely provided by the balance of Casimir attractive and repulsive interactions. Our further control experiment confirms that the impact of residue charge is negligible (*29*).

Such nonmonotonic Casimir interaction acting on the gold nanoplate results from competition between repulsive forces from the Teflon film and attractive forces from the gold substrate. At small separations, the distance between the gold nanoplate and the Teflon surface is shorter than the distance between the nanoplate and the gold substrate. As a result, the repulsive forces contributed by the Teflon film become predominant, and the Casimir interaction energy thus converges to the repulsive interaction between the gold nanoplate and the Teflon film (green dashed line in Fig. 3A). At large separations, the distances from the gold nanoplate to the Teflon surface and from the gold nanoplate to the gold substrate are comparable. Considering that the gold substrate possesses much higher refractive indices than that of Teflon, the attractive forces from the gold substrate are much stronger than the repulsive forces from the Teflon film, and the Casimir interaction energy thus converges to the attractive interaction between the gold nanoplate and the gold substrate (blue dashed line in Fig. 3A).

The underlying physics can be further understood by investigating Casimir force contributions from different frequencies and different parallel momenta that are two integration variables in Casimir force calculation (eq. S1). The repulsive contributions are mainly from large parallel momenta and high frequencies (Fig. 3B). Here, the fluctuation-induced electromagnetic fields are evanescent surface waves, which decay exponentially away from the surface. The larger parallel momentum and larger frequency indicate that the evanescent field decays faster (*30*). If the decaying field cannot penetrate through the Teflon to interact with the gold substrate underneath, this portion of the fluctuation-induced electromagnetic field will interact only with Teflon, thus

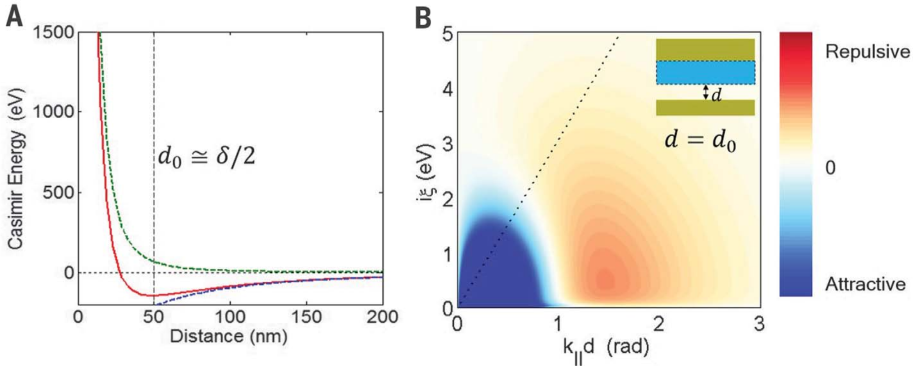

**Fig. 3. Theoretical explanation of the Casimir equilibrium.** (**A**) Casimir interaction energy between the gold nanoplate and the 91-nm-thick Teflon-coated gold surface (red line), between the gold nanoplate and the Teflon film only (green dashed line), and between the gold nanoplate and the gold substrate (blue dashed line). The vertical dashed line labels the equilibrium distance ($d_0$), which is roughly equal to half of the Teflon thickness (δ). (**B**) Casimir force contribution from different frequencies and different parallel momenta ($k_{||}$, the parallel component of the wave vector) at the equilibrium distance denoted in (A). The dashed line is the light line of electromagnetic waves in ethanol. $\xi = -i/\omega$, which describes the frequency along the imaginary axis.

contributing repulsive forces to balance the attractive force from the gold substrate (fig. S8).

We have observed that the Casimir interaction between a gold nanoplate and a Teflon-coated gold surface is repulsive at short separations and attractive at long separations. With the demonstrated Casimir equilibrium, two adjacent objects can remain separated at a well-controlled distance. This fundamental quantum trap is passive, which will lead to contact-free nanomechanical systems and controlled self-assembly.

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## ACKNOWLEDGMENTS

R.Z. thanks J. Pendry for helpful theoretical discussion at Imperial College London before he joined UC Berkeley. **Funding:** This work was primarily supported by the U.S. Office of Naval Research (ONR) MURI program (grant N00014-17-1-2588), the King Abdullah University of Science and Technology Office of Sponsored Research (OSR) (award OSR-2016-CRG5-2950-03), and the Gordon and Betty Moore Foundation. We also acknowledge the AFM user facility at the Molecular Foundry, supported by the U.S. Department of Energy, Office of Science, Office of Basic Energy Sciences (contract DE-AC02-05CH11231). **Author contributions:** R.Z., S.Y., and X.Z. conceived the project. R.Z. performed theoretical investigations. R.Z., L.L., S.Y., and W.B. designed the trapping experiments. R.Z., L.L., and S.Y. performed the trapping measurements. R.Z., L.L., S.Y., and W.B. fabricated the samples. S.Y. performed the zeta potential measurement. W.B. and Y.X. performed AFM measurements with assistance from P.A. All authors contributed to manuscript preparation and discussion. X.Z. and Y.W. guided the research. **Competing interests:** The authors declare no competing interests. **Data and materials availability:** All data are available in the manuscript or the supplementary materials.

## SUPPLEMENTARY MATERIALS

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Materials and Methods
Figs. S1 to S10
Table S1
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Movie S1

20 February 2019; accepted 17 May 2019
10.1126/science.aax0916

Science

# Stable Casimir equilibria and quantum trapping

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*Science* **364** (6444), 984-987.
DOI: 10.1126/science.aax0916

**Something repulsive in the Casimir effect**

Two uncharged objects (metal plates for instance) will experience an attractive force between them, the magnitude of which increases as they are brought closer together. This force, or Casimir effect, is caused by vacuum fluctuations of the electromagnetic field. Effectively, more modes outside than between the objects results in the objects being pushed together. Zhao *et al.* show that the extent of the electromagnetic fluctuations can be controlled by coating one of the objects with a dielectric (Teflon), which changes the Casimir effect to a repulsive force at small distances. This then cancels out the force between plates and produces a point of stable equilibrium.

*Science*, this issue p. 984

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