Nonlinear Quantum Optics in a Waveguide: Distinct Single Photons Strongly Interacting at the Single Atom Level

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We propose a waveguide-QED system where two single photons of distinct frequency or polarization interact strongly. The system consists of a single ladder-type three level atom coupled to a waveguide. When both optical transitions are coupled strongly to the waveguide's mode, we show that a control photon tuned to the upper transition induces a π phase shift and tunneling of a probe photon tuned to the otherwise reflective lower transition. Furthermore, the system exhibits single photon scattering by a classical control beam. Waveguide-QED schemes could be an alternative to high quality cavities or dense atomic ensembles in quantum information processing.

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Strong nonlinear interactions between two distinct optical signals at the few photon level are of foremost importance in quantum information science [1]. The ultimate goal is to build quantum gates to control the transmission and phase of a single photon with another photon at a different frequency or polarization. While resonant interactions in dense atomic ensembles [2–4] exhibit nonlinearities considerably larger than those of conventional materials, they require a large number of atoms with long lifetime dark states at cold temperatures to control single photons [5]. Meanwhile, cavity-enhanced QED systems can enable nonlinear interactions at the single atom level [6], which has led to the demonstration of photon blockade [7–9] and the proposal of single photon switches [10]. Recently, a number of authors identified that a two-level atom coupled to a waveguide mode can induce strong interactions between degenerate photons [11,12]. Unlike cavity-based enhancements, which control photon interactions at discrete modes, waveguide-based enhancements access a one dimensional continuum of reflection and transmission modes, making them ideal for nonlinear frequency mixing of single photons. While waveguide QED has also been proposed to control photon transmission by the quantum state of a Λ-type atom [13], the scheme relies on a classical control beam to realize a single photon switch.

In this Letter we propose an elementary waveguide-QED scheme to achieve a nonlinear interaction of two frequency or polarization distinct single photons at the ultimate limit. The interaction can occur at a single ladder- or V-type three level atom, where here, we consider the ladder configuration. Λ-type atoms [14] are not suitable because the shared upper state inhibits simultaneous strong coupling of both optical transitions to the waveguide. Figure 1 shows a schematic of the photon scattering processes and the atom's energy level diagram. A stream of waveguide photons with two different frequencies (polarizations) are applied in resonance with the atomic transitions. The lower atomic transition $|a\rangle \rightarrow |b\rangle$ is used as the quantum probe channel, whereas the upper transition $|b\rangle \rightarrow |c\rangle$ is used as the quantum control channel. When the atom is initially prepared in the ground state $|a\rangle$, in the absence of the control photon b, photon a will be reflected by the atom. However, in the presence of the control photon b, cascaded two-photon excitation and emission is allowed, which can result in the transmission of photon a through the reflective barrier. The distinguishing feature of waveguide QED is the strong resonant interference of scattered and incident photons by the atomic transition of a single atom [15]. Strong coupling between the atom and the waveguide mode dominates over coupling into other radiative or nonradiative channels and ensures that the incident and scattered waves have the same amplitude, inducing strong interference. If two transitions of the same atom are strongly coupled to the waveguide mode, not only single photon, but also two-photon cascade emission can induce strong interference of the scattered waves, resulting in strong intensity correlation patterns.

In order to understand the physics of this two-photon scattering process, we analyze the second order intensity

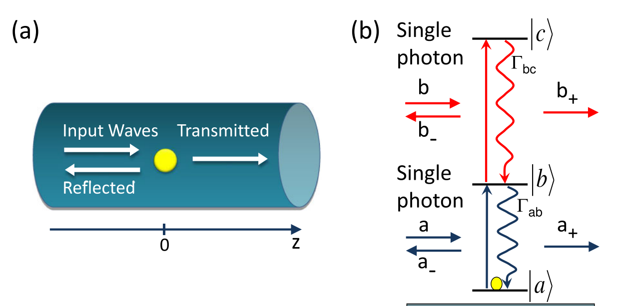

FIG. 1 (color online). Two-photon scattering on a ladder-type atom coupled to waveguide. (a) Schematic of the system. (b) Energy level diagram.

correlations for transmitted and reflected photons. We show that when both atomic transitions are strongly coupled to waveguide modes, photon a will be transmitted on the condition that photon b is also transmitted, which manifests conditional photon tunneling. Moreover, we show that reflected photon a acquires an extra π phase shift if photon b is also reflected, which could be used for quantum logic.

We proceed to calculate the two-photon intensity correlation functions. The Hamiltonian for the scheme shown in Fig. 1, in the interaction picture, is given by

$$V = \hbar\sum_k g_k^* \hat{\sigma}_{cb}\hat{a}_k e^{-i\Delta_k t} + \hbar\sum_q g_q^* \hat{\sigma}_{bc}\hat{a}_q e^{-i\Delta_q t} + \text{H.c.} \tag{1}$$

Here g_k and g_q are single photon coupling coefficients, $\Delta_k = \nu_k - \omega_{bc}$ and $\Delta_q = \nu_q - \omega_{ab}$ are frequency detunings, $\hat{a}_k$ and $\hat{a}_q$ are the annihilation operators, ν_k and ν_q are the frequencies of the photons with wave numbers k and q. $\hat{\sigma}_{ij}$ is the atomic flip operator, ω_{ij} is the frequency of the $|i\rangle \to |j\rangle$ transition.

We assume that the atom is initially prepared in the ground state $|a\rangle$, and the electromagnetic field has single photons in modes k_0 and q_0, so that $A_{kq}(t=-\infty) = 1_{k,k_0}1_{q,q_0}$. The corresponding atom-field state is $|\Psi(t)\rangle = C(t)|0, c\rangle + \sum B_k(t)|1_k, b\rangle + \sum A_{kq}(t)|1_k, 1_q, a\rangle$.

The Schrödinger equation for the probability amplitudes $A_{kq}(t)$, $B_k(t)$, and $C(t)$ is given by

$$\begin{aligned}
\frac{\partial}{\partial t}C &= -i\sum_k g_k^* B_k e^{-i\Delta_k t}\\
\frac{\partial}{\partial t}B_k &= -ig_k C e^{i\Delta_k t} - i\sum_q g_q^* A_{kq} e^{-i\Delta_q t}\\
\frac{\partial}{\partial t}A_{kq} &= -ig_q B_k e^{i\Delta_q t}.
\end{aligned} \tag{2}$$

In order to determine the state of the atom and radiation field, we solve this set of equations analytically using the Weisskopf -Wigner approximation

$$\begin{aligned}
A_{kq}(t) = A_{kq}(-\infty) &- \int_{-\infty}^{t} dt' g_q \sum_{q'} \frac{g_{q'}^* A_{kq'}(-\infty)\exp(i(\nu_q - \nu_{q'})t')}{-i\Delta q' + \frac{\Gamma ab}{2}}\\
&+ \int_{-\infty}^{t} dt' g_q g_k \sum_{k'q'} \frac{g_{q'}^* g_{k'}^* A_{k'q'}(-\infty)\exp(i(\nu_q + \nu_k - \nu_{q'} - \nu_{k'})t')}{(-i\Delta_{q'} + \frac{\Gamma ab}{2})(-i\Delta_{q'} - i\Delta_{k'} + \frac{\Gamma bc}{2})(-i\Delta_{q'} - i\Delta_{k'} + i\Delta_k + \frac{\Gamma ab}{2})}.
\end{aligned} \tag{3}$$

Here Γ_{ij} is the spontaneous decay rate of the $|i\rangle \to |j\rangle$ transition. Three terms are readily identifiable on the right-hand side of Eq. (3): the first represents the input state; the second describes single photon scattering on the $|a\rangle \to |b\rangle$ transition; and the third describes energy conserving two-photon excitation and emission processes.

Next, we proceed to calculate the two-photon wave function $\Psi_{xy}(t, z_b, z_a) = \langle 0, a|\hat{a}_y(t, z_a)\hat{b}_x(t, z_b)|\Psi(t)\rangle$ at two detectors at positions z_a and z_a for time t. Here, the mode indices x or y denote the forward "+" or backward "−" propagation direction, $\hat{b}_\pm(t,z) = \sum_{\pm k>0}\hat{a}_k e^{-i\nu_k t + ikz}$ and $\hat{a}_\pm(t,z) = \sum_{\pm q>0}\hat{a}_q e^{-i\nu_q t + iqz}$ are electric field operators for photons a and b, propagating in the + or − direction. In general, the two-photon wave function for two transmitted photons is given by

$$\begin{aligned}
\Psi_{++}(t, z_a, z_b) &= e^{-i\nu_{k_0}t + ik_0 z_b} e^{-i\nu_{q_0}t + iq_0 z_a}\\
&\times\left(1 + r_{q_0} - r_{q_0}\frac{\frac{\Gamma'_{bc}}{2}\theta(\tau)e^{i\Delta_{q_0}\tau - (\Gamma_{ab}/2)\tau}}{-i\Delta_{q_0} - i\Delta_{k_0} + \frac{\Gamma_{bc}}{2}}\right),
\end{aligned} \tag{4}$$

where Γ'_{ij} is the spontaneous decay rate of the $|i\rangle \to |j\rangle$ transition into the waveguide mode, $\theta(\tau)$ is the Heaviside step function, $\tau = z_b/V_b - z_a/V_a$ is the propagation time difference, $V_{a,b}$ is the group velocity of photon a or b, and $r_{q_0} = -\frac{\Gamma'_{ab}}{2}/(-i\Delta_{q_0} + \frac{\Gamma_{ab}}{2})$ is a single photon reflection coefficient for the $|a\rangle \to |b\rangle$ transition.

Now we consider the case when the incoming photons are in resonance with the atomic transitions. For the stationary process we obtain the two-photon intensity correlation function as $G^{(2)}_{xy}(\tau) = |\Psi_{xy}(t, z_b, z_a)|^2$ where

$$G^{(2)}_{++}(\tau) = (1 - \beta_{ab} + \beta_{ab}\beta_{bc}\theta(\tau)e^{-(\Gamma_{ab}/2)\tau})^2, \tag{5a}$$

$$G^{(2)}_{+-}(\tau) = (-\beta_{ab} + \beta_{ab}\beta_{bc}\theta(\tau)e^{-(\Gamma_{ab}/2)\tau})^2, \tag{5b}$$

$$G^{(2)}_{--}(\tau) = G^{(2)}_{-+}(\tau) = (\beta_{ab}\beta_{bc}\theta(\tau)e^{-(\Gamma_{ab}/2)\tau})^2. \tag{5c}$$

Here $\beta_{ij} = \Gamma'_{ij}/\Gamma_{ij}$ is the spontaneous emission coupling factor to a waveguide mode for the $|i\rangle \to |j\rangle$ transition.

Figure 2 shows the second order intensity correlations for transmitted and reflected photons. These correlations are a result of Fano-type interference between photon pairs interacting with the atom via three possible pathways as shown in Fig. 2(a). In the first pathway the atom remains unperturbed and both incident photons are transmitted, corresponding to the unity term in Eq. (5a). In the second pathway the atom's lower transition is excited by incident photon a while photon b is transmitted without change. The scattered photon has amplitude β_{ab} with a π phase

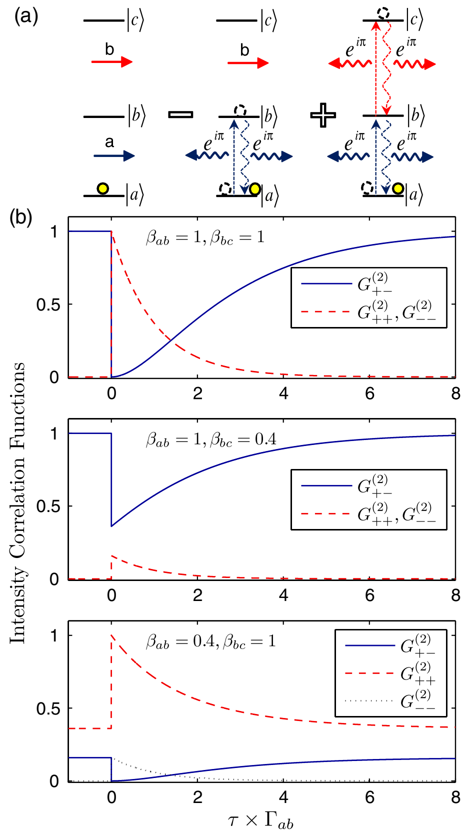

FIG. 2 (color online). (a) The interference of two-photon scattering processes for an atom in waveguide. The vertical arrows denote the atomic excitation and emission processes. The filled and empty dots indicate the final and intermediate atomic states, respectively. The horizontal straight and wavy arrows denote input and scattered photons, respectively. The large "+" and "−" signs indicate constructive and destructive interference. (b) Two-photon intensity correlations between transmitted or reflected photons $b_\pm(t)$ and $a_\pm(t+\tau)$, calculated for different β_{ab} and β_{bc} factors. If $\beta_{ab}=\beta_{bc}=1$, strong photon interaction results in the forward scattering of photon a immediately after transmitted photon b_+ is detected.

shift relative to the incident wave. This process corresponds to the second term in Eq. (5a) and the first term in Eq. (5b). In the limit $\beta_{ab}\rightarrow 1$, the incident and scattered photons interfere and completely suppress uncorrelated transmission of photon a. In the third pathway, both incident photons excite the atomic cascade $|a\rangle\rightarrow|b\rangle\rightarrow|c\rangle$ followed by correlated photon pair emission. The two-photon correlation has an amplitude equal to $\beta_{ab}\beta_{bc}$ that persists for time delays up to the atomic-dephasing time as given by the last terms of Eqs. (5a)–(5c). Since photon b also acquires a π phase shift, photon pair cascade emission has a total 2π phase shift. It can also be viewed as if emitted photon a acquires an extra π phase flip on the condition that transmitted or reflected photon b is detected. This phase flip leads to rich correlations from the interference of the three pathways, shown in Fig. 2(b). Specifically, when photon b is transmitted and $\beta_{bc}\rightarrow 1$, photon a's reflection probability is very low during the time period $1/\Gamma_{ab}$. In the limit $\beta_{ab}\beta_{bc}\rightarrow 1$, photons a and b both have a high probability of being transmitted together. We conclude that this system is a conditional single photon switch.

Figure 2(b) also shows the correlation functions under nonoptimal coupling conditions. Low $\beta_{bc}<1$ reduces the probability of conditional photon a transmission and increases its reflection. Moreover, low $\beta_{ab}<1$ introduces a large background to $G^{(2)}_{++}(\tau)$ due to uncorrelated transmission of photon a by the atom. These effects degrade the performance of the system as a single photon switch.

In order to broaden our understanding of the scattering processes under strong coupling to waveguide mode, we also consider single photon scattering on one of the atomic transitions in the presence of a classical control beam driving the other transition as shown in Fig. 3. We note that the absorption at a quantum dot in the weak coupling regime has been observed before [16]. First, we consider the case shown in Figs. 3(a) and 3(b). Figure 3(b) shows the transmission of a single photon as a function of its frequency detuning (Δ_{q_A}). We take $\Gamma_{bc}=\Gamma_{ab}$ and $\beta_{ab}=1$,

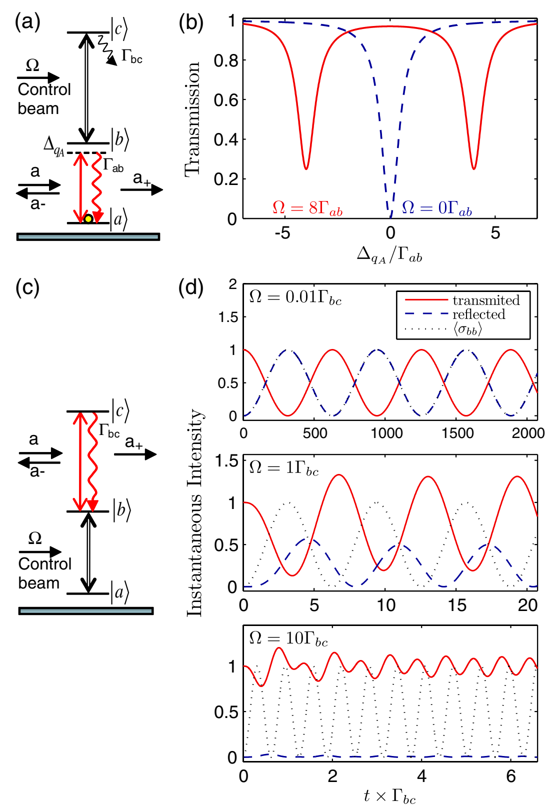

FIG. 3 (color online). (a),(b) EIT-type single photon switch. (a) Energy level diagram. (b) Transmission vs frequency detuning of single photon a. (c),(d) Intensity modulation on a single atom. (c) Energy level diagram; the control and single photon beams are resonant with the lower and upper transitions. (d) Instantaneous intensities for transmitted and reflected photons and occupational probability $\langle\sigma_{bb}\rangle$ of state $|b\rangle$ as a function of time at different Ω.

to ensure high fidelity switching characteristics. Similar to the classical EIT scheme for dense atomic ensembles, a strong control beam, i.e., $|\Omega|^2 > \Gamma_{ab}\Gamma_{bc}$, tuned in resonance with the $|b\rangle \rightarrow |c\rangle$ transition creates a transparency window for a single photon. Here and thereafter, Ω is the Rabi frequency of the control beam. In order to avoid intensity saturation effects, the incident single photon rate should be much smaller as compared to Γ_{ab}.

A quite different situation occurs when the control and single photon transitions switched [Fig. 3(c)]. We assume that $\beta_{bc} = 1$ and the atom is prepared in state $|a\rangle$ at $t = 0$. Figure 3(d) shows the instantaneous intensities for transmitted and reflected signals and the occupational probability ($\langle\sigma_{bb}\rangle$) of state $|b\rangle$ as a function of time calculated for different $\Omega \gg \Gamma_{ab}$. The transient response of this system is characterized by the two atom-field coupling strengths: the atom flopping frequency of the lower transition Ω for the strong control beam; and the field-atom energy exchange rate at the upper transition Γ_{bc} for the single photon. The control beam modulates the population of state $|b\rangle$, leading to a periodic modulation of upper state spontaneous emission. If the lower transition is driven slowly, i.e., $\Omega \ll \Gamma_{bc}$, the reflection function follows the atomic population in state $|b\rangle$, resulting in 100% modulation of the input signal. However, when $\Omega \gg \Gamma_{bc}$, photon scattering is suppressed as the occupation probability of state $|b\rangle$ oscillates faster than the single photon-atom coupling rate. Therefore the transmission curve shows only small oscillations around unity. We also note that although the cycle-average scattered intensity is equal to the incident one, the transmitted instantaneous intensity can be higher than 100% due to energy exchange between the classical and quantum beams through the atom and relatively slow energy circulation on the upper transition.

This story would not be complete without a discussion of experimental implementation. The proposed schemes require a robust ladder-type atom and a waveguide confinement mechanism to achieve a strong atom-field coupling. Ladder-type atoms are readily available in technologically relevant solid states systems, for example, as a subset of the four level fine structure of InAs/GaAs quantum dots [16]. While V-type systems [17] are also viable and reveal similar single photon switching behavior, the ladder-type configuration of InAs/GaAs quantum dots is attractive due to large (several nm) wavelength separation between biexciton transitions, which is crucial for efficient frequency filtering. For the slow modulation case of Fig. 3(d) NV color centers in diamond can be used as they have a long lifetime microwave transition cascaded with an optical one. In choosing a suitable atomic system it is important to account for additional dephasing mechanisms beside spontaneous decay that can adversely affect the amplitudes and phase of both uncorrelated and correlated single photon scattering. Strong atom-field coupling within a waveguide environment can take advantage of a number of schemes proposed recently. For example, β factors as high as 85% can be obtained in photonic crystal waveguides [18]; and plasmonic waveguides promise even stronger coupling [19–21], provided the problem of emission quenching near metal surfaces can be avoided. However, even tightly focused beams [22,23] are sufficient to yield high β factors, so elaborate schemes could be avoided. Approaches using semiconductor waveguides, compatible with quantum dot technologies, are particularly promising. For example, quantum dots with radial dipole orientation at the center of a dielectric nanowire waveguide [24,25], would allow β factors of 90% [26] and slot waveguides [27], give even higher β factors reaching 96%, even when averaged over all dipole orientations [28].

In summary, we showed that strong coupling of multiple atomic transitions to waveguide enables strong interaction of distinct single photons. We believe the proposed schemes open new directions to engineer photonic quantum logic devices without the use of high Q and small mode volume cavities or dense atomic ensembles.

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