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## Low-loss and energy efficient modulation in silicon photonic waveguides by adiabatic elimination scheme

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High-speed Silicon Photonics calls for solutions providing a small footprint, high density, and minimum crosstalk, as exemplified by the recent development of integrated optical modulators. Yet, the performances of such modulators are hindered by intrinsic material losses, which results in low energy efficiency. Using the concept of Adiabatic Elimination, here, we introduce a scheme allowing for the low-loss modulation in densely packed waveguides. Our system is composed of two waveguides, whose coupling is mediated by an intermediate third waveguide. The signal is carried by the two outer modes, while the active control of their coupling is achieved via the intermediate dark mode. The modulation is performed by the manipulation of the central-waveguide mode index, leaving the signal-carrying waveguides unaffected by the loss. We discuss how Adiabatic Elimination provides a solution for mitigating signal losses and designing relatively compact, broadband, and energy-efficient integrated optical modulators. *Published by AIP Publishing*. [http://dx.doi.org/10.1063/1.4994024]

The past decade has seen major advances in Silicon Photonics with the demonstration of CMOS-compatible integrated modulators, which operate at low power with micron-scale footprints. Most of these devices physically rely on silicon nonlinear optical processes, such as Raman scattering, self- and cross-phase modulation, and four-wave mixing.<sup>1-3</sup> Although silicon has excellent linear and nonlinear optical properties (e.g., high linear and nonlinear refractive indices), the performances of the aforementioned devices are ultimately limited by nonlinear material losses. In this paper, we demonstrate that Adiabatic Elimination (AE)-recently introduced in the context of waveguides<sup>4</sup>---offers a unique route to minimize losses in optical modulation. By decoupling the signal-carrying waveguides and the control waveguide, the AE offers an approach that effectively reduces the nonlinear losses while maintaining an efficient nonlinear index modulation over a large bandwidth in a relatively small footprint. We first describe the effect of nonlinear losses in the AE scheme. Then, we discuss the time-dependent nonlinear model employed to study these losses considering the Free Carrier (FC) generation and absorption as well as the nonlinear losses induced by the Kerr effect. We further explain how the different degrees of freedom of AE waveguide systems allow for the optimization of performances in terms of both modulation speed and depth. Finally, a comparison with existing modulation schemes is presented.

In modulators, the signal and the control typically share the same waveguide. For this reason, the nonlinear losses induced by the control directly affect the signal. It is therefore desirable to decouple the control waveguide from the signal-carrying waveguide(s). In that regard, we consider a planar arrangement of three waveguides, where the signal evolves in the two outer waveguides (Fig. 1). The different coupling and phase mismatch satisfy the AE condition

$$\Delta\beta_{12}|, |\Delta\beta_{23}| \gg V_{12}, V_{23}, \tag{1}$$

where  $\Delta\beta_{ij}$  stands for the phase mismatch between waveguides *i* and *j* and  $V_{ij}$  stands for the corresponding coupling.



FIG. 1. Schematic of the Adiabatic Elimination modulation scheme where three collinear waveguides are packed beyond the diffraction limit. This results in strong couplings between all the waveguides (all are strongly coupled). In this regime, in addition to strong couplings, there must be a strong detuning between the waveguide modes such as  $|\Delta\beta_{12}|, |\Delta\beta_{23}| \gg V_{12}, V_{23}$ , with  $\Delta\beta_{ij}$  being the phase mismatch between waveguides *i* and *j* and  $V_{ij}$ being the coupling between them. Once the AE regime is reached, the middle waveguide is effectively decoupled from the outer waveguides. However, controlling the mode index of the decoupled middle waveguide affects the propagation dynamics of a signal propagating back and forth between the outer waveguides and thus modulates the output power out of waveguides 1 and 3. The mode index control mechanism in the middle waveguide can be electrical, optical, thermal, or mechanical.

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In contrast to conventional waveguide modulators, the losses in the AE scheme are mainly induced by the control applied only on the middle waveguide, which supports a dark mode.<sup>4</sup> Therefore, the control-induced losses do not affect the signal, which is carried by the two outer modes. Moreover, since the modulation is achieved through the manipulation of the middle-waveguide-mode refractive index, the control mechanism can independently be electrical, thermal, optical, or mechanical. Without the loss of generality, here, the control is achieved through the propagation of an optical pulse in the middle waveguide.

The real part of the refractive index is modulated by the Kerr effect originating from silicon's third order nonlinearity,  $\chi^{(3)}$ , and the loss is induced by Two Photon Absorption (TPA) and Free Carrier Absorption (FCA). We model the propagation of the control pulse by a modified Non-Linear Schrödinger Equation (NLSE)<sup>5,6</sup>

$$\frac{\partial A}{\partial z} + \frac{\alpha_l}{2}A = i\kappa |A|^2 A - \frac{\alpha_f}{2}A, \qquad (2)$$

where *A* is the slowly varying amplitude,  $\alpha_l$  the linear loss,  $\kappa$  the nonlinear coefficient, and  $\alpha_f$  the TPA loss. In Eq. (2), the effects of dispersion are neglected. Indeed, if the group velocity dispersion in silicon wires can be significantly enhanced compared to bulk silicon, it does not mean that a sensible temporal distortion is induced over a short distance.<sup>7</sup> In our case, we estimate that a 140 fs-long pulse propagating over 300  $\mu$ m is only broadened by 150 fs.<sup>4</sup> The nonlinear coefficient in Eq. (2) reads  $\kappa = \frac{n_2k_0}{A_{eff}} + i\frac{\Gamma}{2}$ , where  $n_2 = 2 \times 10^{-18} \frac{m^2}{W}$  is the Kerr coefficient and  $\Gamma$  is derived from the TPA coefficient  $\beta_{TPA} = A_{eff}\Gamma = 5 \times 10^{-12}m/W.^{5.6}$  The effective area of the middle-waveguide mode,  $A_{eff}$ , is defined as<sup>6</sup>

$$A_{eff} = \frac{\left(\int \int |E_y|^2 dx dy\right)^2}{\int \int |E_y|^4 dx dy}.$$
(3)

In the following, we consider a silicon buried in the silica substrate middle-waveguide of width 275 nm and height 340 nm, which sustains a TM mode whose effective area is estimated from Eq. (3) to be  $A_{eff} = 0.11 \mu m^2$ . FCA is included by  $\alpha_f = \sigma N$ , where  $\sigma = 1.45 \times 10^{-21} m^2$  for silicon<sup>6</sup> and N is the carrier density produced by TPA. The carrier density is related to the field amplitude by<sup>6</sup>

$$\frac{\partial N}{\partial t} = \frac{\beta_{TPA}}{2hv_0} \frac{|A|^4}{A_{eff}^2} - \frac{N}{\tau_c},\tag{4}$$

where  $\tau_c$  is the effective carrier lifetime ( $\tau_c = 0.5$  ns).<sup>6,8</sup> For a repetition rate (80 MHz) lower than  $1/\tau_c$ , the free carriers mostly recombine between two pulses. This simplification reduces the problem to a single-pulse propagation. Combining Eqs. (2) and (4), we plot in Fig. 2(a) the pulse propagation for an input peak power of 400 W. The output pulse matches the input-pulse shape, meaning that the long-lived FCA effect is negligible. A weak FCA is further confirmed in Fig. 2(b), showing the evolution of the FC density, N during the pulse propagation. At any time and for any propagation length, Fig. 2(b) shows that the FC density induced by the pulse is several orders of magnitude lower than the typical carrier



FIG. 2. (a) Results from the modified NLSE showing no temporal distortion due to FC after propagation of the 400 W peak power pump pulse in the middle waveguide (blue curve: pump beam at the input; red curve: pump beam at the output). (b) Spatio-temporal dependence of the pump-induced free carrier concentration along a  $300 \,\mu$ m long Silicon waveguide. It is apparent that the FC is significantly negligible compared to sensible concentrations used to modulate the refractive index.

concentrations needed to affect tangibly the index of refraction of silicon  $(10^{17}-10^{18} \text{ cm}^{-3} \text{ (Ref. 9)} \text{ compared to } 10^{11} \text{ cm}^{-3}$ here). Therefore, under previously reported experimental conditions, the fast Kerr nonlinearity dominates the slow FC effect. It should be noted that the nonlinear Kerr effect is considered instantaneous because it relies on bound electrons, and therefore, the all-optical modulation can be ultrafast (as already been observed in Ref. 4). However, should a fast (>1 GHz) electronic modulation be desired, considerations such as the capacitance of the electrodes and travelling-wave configuration have to be taken into account as in any modulator, which are beyond the scope of the present work.

Having determined the mechanism at the origin of the modulation, we can also evaluate the loss expected from an optical control performed through the Kerr effect. The NLSE [Eq. (2)] allows us to evaluate the attenuation induced by the imaginary part of the refractive index  $\kappa$ 

$$\ln\left(\frac{I_{out}}{I_{in}}\right) = -\frac{4\pi\kappa}{\lambda}l_{prop}.$$
(5)

We deduce from Eq. (5) that the loss induced by the control pulse in the middle waveguide is  $\text{Im}(n) = \kappa = 4.6 \times 10^{-5}$ .

The AE condition is also satisfied when the middle waveguide is lossy either because of linear losses originating from the imaginary part of the refractive index (e.g., plasmonic and near resonant interactions) or because of nonlinear losses originating from the imaginary part of nonlinear susceptibility (e.g., TPA in Silicon Photonics). In that regard, the AE modulation scheme offers a route to loss tolerant modulation. When losses are present in the middle waveguide, the corresponding mode index is modified to include an imaginary part, while the signal-carrying mode indices remain real:  $\beta_1 = \frac{2\pi}{\lambda} n_1^{eff}, \beta_2 = \frac{2\pi}{\lambda} n_2^{eff} + i\kappa_2^{eff}, \beta_3 = \frac{2\pi}{\lambda} n_3^{eff}$ . Nevertheless, since the AE minimizes the signal propagation in the middle waveguide, it leads to a dramatic reduction in the effective signal loss coefficient by several orders of magnitude.

To show this behavior, we compare two cases. In the first case, we consider a three waveguide AE configuration, where the imaginary part of the middle-waveguide refractive index is set to  $\kappa_2 = 4.6 \times 10^{-5}$  as estimated in the previous paragraph. The second case is a two-waveguide coupler where the coupling yields an equivalent inversion length compared to the first AE case. Although the losses considered here are induced by the strong control pump through TPA and FCA, the analysis presented here is valid for other loss mechanisms such as metallic contacts or plasmonic waveguides. In a numerical analysis carried out using an EigenMode Expansion (EME) method<sup>10</sup> on COMSOL, we have simulated the propagation of a signal injected in waveguide #1 (outer waveguide see Fig. 1). In the two-waveguide case, the loss is considered in both waveguides since the intense pump oscillates between the 2 waveguides and induces TPA and FCA in both waveguides. For the three-waveguide AE case, the loss is considered in the middle waveguide, as the intense pump propagates only along the middle waveguide.

As shown in Figs. 3(a) and 3(b), the effective loss experienced by the signal in the three-waveguide AE configuration is significantly reduced compared to the standard two-waveguides coupler (0.28 dB compared to 6.88 dB), despite the fact that the same loss coefficient is considered in both cases. The origin of the lower effective loss is related to the nature of the signal propagation in the outer waveguides in the AE scheme. The rate of light transfer to the middle waveguide is equal to the rate of transfer from the middle waveguide, meaning that the effective signal propagation there is significantly reduced. For this reason, the AE of the lossy middle waveguide dramatically reduces the overall loss of the modulator while still allowing coupling control through the middle waveguide.

Several degrees of freedom available in the AE scheme, namely, the direct couplings  $V_{12}, V_{23}$ , the indirect coupling  $V_{13}$ , and the phase mismatches  $\Delta\beta_{12}, \Delta\beta_{23}, \Delta\beta_{13}$ , allow for the tuning of the effective coupling between the outer waveguides. This effective coupling can be exploited to improve the extinction ratio and to reduce the power consumption. A



FIG. 3. Tolerance to the induced loss in an AE three-waveguide configuration. (a) Simulation of propagation in a three-waveguide AE configuration including the loss of  $\text{Im}(n) = \kappa = 4.6 \times 10^{-5}$  in the middle waveguide as evaluated from the generalized NLS model. We observe very minute signal loss since the propagation length of the signal in the middle waveguide is significantly reduced. As a comparison, an equivalent two-waveguide system is calculated under the same conditions (b).

particularly promising direction resides in the design of a device with a middle waveguide near its cutoff. By doing so, the device operates in the regime where the signal rapidly oscillates between the outer waveguides and then becomes much more sensitive to the index change in the middle control waveguide. To exemplify this approach, we have simulated a device where the outer waveguides have the same dimensions ( $W_1 = W_3 = 220 \text{ nm}$ ), whereas the middle waveguide is near the cutoff ( $W_2=170$  nm). In order to meet the right condition  $(V_{13} > \frac{V_{12}^2}{\Delta\beta})$ , we reduce the overall gap (between waveguide 1 and waveguide 3) to 470 nm to increase the  $V_{13}$  coupling. The signal injected in waveguide #1 oscillates quickly between #1 and #3 as shown in Fig. 4. Fast oscillations indicate that the device is more sensitive to index changes in the middle waveguide. Our simulation shows that in this configuration, an extinction ratio of 20 dB is achieved with a change of index of  $\Delta n = 9 \times 10^{-4}$  requiring only 8 pJ (power consumption 9 times lower than that reported in Ref. 4). A further improved performance, such as a higher extinction ratio with sub pJ consumption, can be achieved with a dedicated optimization of the degrees of freedom. Additionally, it is worth noting the recent review<sup>11</sup> which identifies clearly key parameters for optoelectronic devices to be able to progress toward the attoJoule regime. A central conclusion of this review is that the device has to be small for a modulator on the order of 0.03  $\mu m^3$ . In our case, replacing the control waveguide (middle) with a thin plasmonic waveguide would allow us to shrink the length of the



FIG. 4. Adiabatic elimination with the middle waveguide near the cutoff. (a) The signal oscillates fast between the outer waveguides because of the increased effective coupling. The fast oscillations make the configuration more sensitive to modulation of the middle waveguide. To demonstrate this point, we compare the modulation performances of the configuration tested in Ref. 4 with the proposed configuration near the cutoff. (b) As we can see, in the configuration used in Ref. 1, the signal oscillates slowly in waveguide 1, and thus, it requires a significant amount of index change in waveguide 2 to achieve a sensible output modulation in waveguide 1. (c) On the other hand, in the configuration near the cutoff, the signal oscillates very fast in waveguide 1 and a slight change of index in waveguide 2 results in a significant modulation of output from waveguide 1.

TABLE I. Comparison between the different modulator architectures demonstrated to date: Resonators, interferometers, and AE. Power consumption, footprint, and operating spectrum are the performance metrics taken into account.

Structure	Mach-Zehnder interferometer	Mach-Zehnder interferometer	Ring	Disk	Ring	Adiabatic elimination
Modulation principle	Depletion of a horizontal p-n junction <sup>13</sup>	Forward biased diode <sup>14</sup>	Forward biased diode <sup>15,16</sup>	Reverse-biased p-n junction <sup>17</sup>	Reverse-biased p-n junction <sup>18</sup>	Potentially depletion of a p-n junction
Device footprint ( $\mu$ m <sup>2</sup> )	$\sim 1 \times 10^4$	$\sim \! 1 \times 10^3$	$\sim \! 1 \times 10^2$	20	$\sim 1 \times 10^3$	$\sim 2 \times 10^2$
Energy per bit (fJ/bit)	$\sim \! 3 \times 10^4$	$\sim 5  imes 10^3$	$\sim 300$	85	50	$\sim 8 \times 10^3$
DC modulation depth/ insertion loss (dB)	>20/~7	6-10/~12	>10/<0.5	8/1.5	6.5/2	20/~2 (potentially around 0.2)
Working spectrum (nm)	>20		$\sim 0.1$	$\sim 0.1$	0.1	>100 nm
Thermal stability	Good	Good	Very poor	Very poor	Very poor	Good

device bringing the active volume of the device to the order of 0.05  $\mu$ m<sup>3</sup>. As pointed above, we emphasize that the AE configuration, by minimizing the amount of optical power transiting in the control waveguide, makes this plasmonic approach feasible with respect to metallic losses.

In terms of performance comparison, we present in Table I an adaptation from Ref. 12, which includes the AE device. We consider our simulated device that offers improved performances, while these can be further optimized. There is a tradeoff between power consumption, optical spectrum, and device footprint. While resonators excel in terms of power consumption and small footprint, they suffer from narrow bandwidths as well as fabrication tolerances. Moreover, the power needed to stabilize them thermally should be further included in the power consumption analysis. In contrast, interferometers have more relaxed fabrication tolerances and wider operating spectra and are less sensitive to temperature variations. Their drawbacks reside in large footprints and large power consumptions. In that regard, the AE modulation scheme outperforms interferometer approaches by offering much reduced insertion losses and smaller footprints. At the same time, while offering a footprint comparable to the ring resonator, the AE modulator is broadband and does not suffer from the thermal stabilization issues, which hinder the development of resonance-based modulators.

In this paper, we have presented a scheme to perform optical modulation based on Adiabatic Elimination (AE), which enables the realization of low-loss and low-energy consumption devices. The AE decouples the signal carrying modes from the control one. As a result, it effectively reduces the nonlinear losses and simultaneously maintains an efficient broadband index modulation in a small footprint. The different degrees of freedom available in the AE based modulators allow for an optimization of the performances in terms of both modulation speed and depth, which opens the way for energy-efficient and ultrafast optical modulators.

The device design was supported by the Office of Naval Research (ONR) MURI program under Grant No. N00014-13-1-0678; the energy efficiency study was supported by the "Light-Material Interactions in Energy Conversion" Energy Frontier Research Center funded by the U.S. Department of Energy, Office of Science, Office of Basic Energy Sciences under Award No. DE-AC02-05CH11231. <sup>1</sup>R. Jones, A. Liu, H. Rong, M. Paniccia, O. Cohen, and D. Hak, "Lossless optical modulation in a silicon waveguide using stimulated Raman scattering," Opt. Express **13**, 1716–1723 (2005).

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